Re-Entry Phase of Ballistic Missile Tracking Estimation using Extended Kalman Filtering

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ABSTRACT:
The tracking of a ballistic missile in its re entry phase is a big defense challenge. The re entry vehicle leaves a quiet exo – atmospheric phase and a quasi – Keplerian motion to an endo – atmospheric phase with large aerodynamic loads and a sudden deceleration. The motion is then non – linear and the evolution of the drag is very difficult to predict. Though the re - entry vehicle tracking is a non linear filtering problem there are several filters that are developed to solve this problem. Several researchers have attempted this problem using Linear squares filtering and Kalman filtering techniques. In this work, Extended Kalman Filtering (EKF) technique is applied for Re-Entry Phase of Ballistic Missile Tracking Estimation and estimation parameters are generated and presented in this paper.

Keywords: Ballistic missile, Kalman filtering, Extended Kalman filtering, Linear square filtering

1. INTRODUCTION

1.1. Tracking
Target tracking are used in many important applications of surveillance, guidance and obstacle avoidance systems. Where the main objectives are to determine position, velocity, acceleration and course of one or more moving targets. In such a tracking system, different types of sensors are needed to give measurements of and/or from the targets. This depends on, if the target is a known friendly passenger aircraft giving measurement information from the aircraft to ground airport, or a hostile aircraft that the only measurements available are from your own surveillance systems at fixed locations or on moving platforms. The sensors giving measurements can be radars, cameras, infrared cameras, sonars etc. The measurements will have some kind of measurement noise depending on the sensor; one sensor may be inaccurate on far range, another more inaccurate on short range giving good measurements on far range.

To handle a complex tracking problem, some kind of a recursive filter for target state estimation is needed. The term “filter” is used because one needs to “filter” out the noise, or eliminate an undesired signal, from the measurements. The Kalman filter is a well-known filter used on linear Gaussian problems. In the later years nonlinear filtering have become more and more in the focus of interest. Since the Kalman filter requires linear system equations, nonlinear systems must be linearized, which leads to the Extended Kalman filter (EKF). Another nonlinear approach is the Sigma Point filter (SPF), also known as Unscented Kalman filter.

1.2 Estimation:
Estimation is a “process of inferring the value of quantity of interest from indirect, inaccurate and uncertain observations”. The observations we make are always uncertain and noisy. They relate to the process that is rarely known with confidence. Further, we very often do not observe the actual quantity of interest, rather the variables we observe only indirectly infer the value of his quantity. An estimate that turns out to be incorrect will be an overestimate if the estimate exceeded the actual result or an underestimate if the estimate fell short of the actual result. Estimation theory deals with finding estimates with good properties. This process is used in signal processing, for approximating an unobserved signal on the basis of an observed signal containing noise.

More rigorously, estimation can be viewed as the process of selecting a point from a continuous space, i.e., best estimate. For estimation of yet-to-be observed quantities, forecasting and prediction are applied. In order to estimate a system, minimum two models are required. The first model is a process model, the model which describes the evolution of the state with time; the dynamic model. That is a differential equation, or a set of differential equations. The second model is the measurement model. This is an algebraic equation with noise. These two models give the system of process model and measurement model.

2. ESTIMATION USING KALMAN BASED ESTIMATORS
The Kalman filter, also known as linear quadratic estimation (LQE), is an algorithm that uses a series of
measurements observed over time, containing noise (random variations) and other inaccuracies, and produces estimates of unknown variables that tend to be more precise than those based on a single measurement alone.

The Kalman filter is most often conceptualized as two distinct phases: "Predict" and "Update". The predict phase uses the state estimate from the previous timestep to produce an estimate of the state at the current timestep. This predicted state estimate is also known as the a priori state estimate because, although it is an estimate of the state at the current timestep, it does not include observation information from the current timestep. In the update phase, the current a priori prediction is combined with current observation information to refine the state estimate. This improved estimate is termed the a posteriori state estimate. The covariance prediction and update equations combined together result in the (discrete time) matrix Riccati equation.

Typically, the two phases alternate, with the prediction advancing the state until the next scheduled observation, and the update incorporating the observation. However, this is not necessary; if an observation is unavailable for some reason, the update may be skipped and multiple prediction steps performed. Likewise, if multiple independent observations are available at the same time, multiple update steps may be performed (typically with different observation matrices $H_k$).

**Predict**

Then the predicted (a priori) state estimate can be obtained as follows:

$$\hat{x}_{k|k-1} = F_k \hat{x}_{k-1|k-1} + B_k u_{k-1}$$

Predicted (a priori) estimate covariance can be obtained as follows:

$$P_{k|k-1} = F_k P_{k-1|k-1} F_k^T + Q_k$$

**Update**

Calculate Innovation or measurement residual

$$\tilde{z}_k = y_k - \hat{H}_k \hat{x}_{k|k-1}$$

Calculate Innovation (or residual) covariance

$$S_k = \tilde{H}_k P_{k|k-1} \tilde{H}_k^T + R_k$$

Calculate Optimal Kalman gain

$$K_k = P_{k|k-1} H_k^T \tilde{S}_k^{-1}$$

Obtain the Updated (a posteriori) state estimate

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \tilde{z}_k$$

Updated (a posteriori) estimate covariance

$$P_{k|k} = (I - K_k H_k) P_{k|k-1} (I - K_k H_k)^T + K_k R_k K_k^T$$

It is a common misconception that the Kalman filter assumes that all error terms and measurements are Gaussian distributed. Kalman's original paper derived the filter using orthogonal projection theory to show that the covariance is minimized, and this result does not require any assumption, e.g., that the errors are Gaussian. He then showed that the filter yields the exact conditional probability estimate in the special case that all errors are Gaussian-distributed. Extensions and generalizations to the method have also been developed which work for non-linear system

- Extended Kalman filter
- Unscented Kalman filter

3. ESTIMATION USING EXTENDED KALMAN FILTER

The Kalman filter equations for estimating the system state $x$ for a linear system where presented. Unfortunately linear systems do not exist in the real world. But some systems may behave close to a linear system over small time intervals so that linear filtering can give satisfactory results. However one may come accross nonlinear systems that do not behave linear in any way, and a non-linear filter approach are necessary. It appears that no particular approximate filter is consistently better than any other, though any nonlinear filter is better than a strictly linear one.

The extended Kalman filter (EKF) is the nonlinear version of the Kalman filter which linearizes about an estimate of the current mean and covariance. The Kalman filter addresses the general problem of trying to estimate the state of a discrete-time controlled process that is governed by a linear stochastic difference equation. But what happens if the process to be estimated and (or) the measurement relationship to the process is non-linear? Some of the most interesting and successful applications of Kalman filtering have been such situations. The extended Kalman filter (EKF) is the nonlinear version of the Kalman filter which linearizes about an estimate of the current mean and covariance.

In engineering most systems are nonlinear, so some attempt was immediately made to apply this filtering method to nonlinear systems, so as to overcome the disadvantage of kalman filter. The EKF which adapted techniques, namely multivariate Taylor Series expansions, from calculus to linearize about a working point became the working solution. If the system model (as described below) is not well known or is inaccurate, then Monte Carlo methods, especially particle filters are employed for estimation. Monte Carlo techniques predate the existence of...
the EKF but are more computationally expensive for any moderately dimensioned state-space. It is known from the theory that the Kalman filter is optimal in case that:

- the model perfectly matches the real system,
- the entering noise is white and
- the covariances of the noise are exactly known.

Several methods for the noise covariance estimation have been proposed during past decades. One, ALS, was mentioned in the previous paragraph. After the covariances are identified, it is useful to evaluate the performance of the filter, i.e., whether it is possible to improve the state estimation quality. It is well known that, if the Kalman filter works optimally, the innovation sequence (the output prediction error) is a white noise. The whiteness property reflects the state estimation quality. For evaluation of the filter performance it is necessary to inspect the whiteness property of the innovations.

### 3.2.1 Formulation:

The time update and the measurement update are implemented in their non-linear forms, but for the covariance matrix, the linearization is needed.

So, in the extended Kalman filter, the state transition and observation models need not be linear functions of the state but may instead be differentiable functions.

\[ \dot{x}_k = f(x_{k-1}, u_{k-1}) + w_{k-1} \]
\[ z_k = h(x_k) + v_k \]

where \( w_k \) and \( v_k \) are the process and observation noises which are both assumed to be zero mean multivariate Gaussian noises with covariance \( Q \) and \( R \) respectively.

The function \( f \) can be used to compute the predicted state from the previous estimate and similarly the function \( h \) can be used to compute the predicted measurement from the predicted state. However, \( f \) and \( h \) cannot be applied to the covariance directly. Instead a matrix of partial derivatives (the Jacobian) is computed.

At each time step, the Jacobian is evaluated with current predicted states. These matrices can be used in the Kalman filter equations. This process essentially linearizes the non-linear function around the current estimate.

\[ \text{Jacobian} = \begin{bmatrix} \frac{\partial f}{\partial x_k} & \frac{\partial f}{\partial u_k} \\ \frac{\partial h}{\partial x_k} & \frac{\partial h}{\partial u_k} \end{bmatrix} \]

### Fig 3.1: Differences between Kalman and Extended Kalman Filter

**Discrete time-predict and update equations**

**Predict**

Predicted state estimate

\[ \hat{x}_{k|k-1} = f(\hat{x}_{k-1|k-1}, u_{k-1}) + w_{k-1} \]

Predicted covariance estimate

\[ P_{k|k-1} = \Phi_{k|k-1}P_{k-1|k-1}\Phi_{k|k-1}^T + Q_k \]

**Update**

Innovation or measurement residual is

\[ \tilde{z}_k = z_k - h(\hat{x}_{k|k-1}) \]

Innovation (or residual) covariance is

\[ S_k = H_kP_{k|k-1}H_k^T + R_k \]

Near Optimal Kalman gain can be calculated as

\[ K_k = P_{k|k-1}H_k^T S_k^{-1} \]

Obtain the Updated (a posteriori) state estimate

\[ \hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k\tilde{z}_k \]

Updated (a posteriori) estimate covariance is

\[ P_{k|k} = (I - K_kH_k) P_{k|k-1} \]

where the state transition(\( \Phi_k \)) and observation matrices are defined to be the following Jacobians

\[ \frac{\partial f}{\partial x_k} = \text{Jacobian} \]

\[ H_k = \frac{\partial h}{\partial x_k} \]

\[ Q_k = I + H_k \Phi_k + \frac{(H_k \Phi_k)^2}{2} + ... \]

\[ R_k = \frac{(\text{Jacobian})^2}{3} + ... \]

### 4. SIMULATION RESULTS

**4.1 Algorithm for Extended Kalman Filter:**

1. Initialize the first estimate and the initial covariance value.
2. Evaluate the Jacobian matrices \( f, h \) with current predicted estimates
3. Predict the value of priori state estimate and priori covariance.
4. Calculate the innovation residual and innovation covariance.
5. Calculate the optimal kalman gain.
6. Update the state estimate \( \hat{x} \) and estimate covariance \( P \).
7. Steps from 2 to 6 are iterated for \( i = \text{no. of measurement times} \)
8. \( \hat{x} \) gives the estimated states and the ‘p’ gives the values of the error covariance.

Position error in ECEF(X)
5. CONCLUSION:
Kalman Filters and Extended Kalman Filter (EKF) produce estimates that are very near to measured values which means both can be used for the estimation of the position of an attacking missile. The EKF implementation results in values that are much close to the real values than those produced by KF estimates and hence more reliable positioning and tracking can be achieved.

The Extended Kalman filter linearizes the nonlinear model through a single point altogether. In addition, if the initial estimate of the state is wrong, or if the process is modeled incorrectly, the filter may quickly diverge, owing to its linearization. Another problem with the extended Kalman filter is that the estimated covariance matrix tends to underestimate the true covariance matrix and therefore risks becoming inconsistent in the statistical sense without the addition of "stabilizing noise"

6. References